Fill in the following identities. **SCORE: _____ / 14 PTS**

[a] NEGATIVE ANGLE IDENTITY:

$\sec(-x) = \sec x$

[b] PYTHAGOREAN IDENTITY:

$\cot^2 x = \csc^2 x - 1$

[c] SUM OF ANGLES IDENTITY:

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

[d] DIFFERENCE OF ANGLES IDENTITY:

$\tan(x-y) = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$

[e] POWER REDUCING IDENTITY:

$\sin^2 x = \dfrac{1-\cos 2x}{2}$

[f] HALF ANGLE IDENTITY:

$\cos\frac{1}{2}x = \pm\sqrt{\dfrac{1+\cos x}{2}}$

[g] DOUBLE ANGLE IDENTITY:
WRITE ALL 3 VERSIONS

$\cos 2x = \cos^2 x - \sin^2 x,\ 2\cos^2 x - 1,\ 1 - 2\sin^2 x$

If $\cos x = \frac{2}{3}$ and $\frac{3\pi}{2} < x < 2\pi$, find the values of the following expressions. **SCORE: _____ / 30 PTS**

Write each final answer as a single fraction in simplest form, including rationalizing the denominator.

[a] $\sin 2x$

$= 2\sin x \cos x$

$= 2\left(-\frac{\sqrt{5}}{3}\right)\left(\frac{2}{3}\right)$

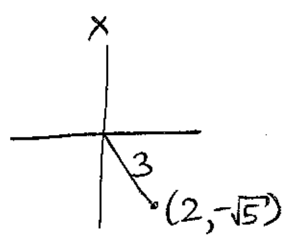
$= -\dfrac{4\sqrt{5}}{9}$

[b] $\cos(\underbrace{\arctan(-2)}_{y} - x)$

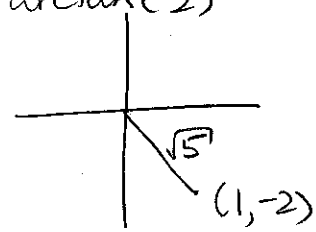
$= \cos y \cos x + \sin y \sin x$

$= \frac{1}{\sqrt{5}}\,\frac{2}{3} + \frac{-2}{\sqrt{5}}\,\frac{-\sqrt{5}}{3}$

$= \dfrac{2+2\sqrt{5}}{3\sqrt{5}} = \dfrac{2\sqrt{5}+10}{15}$



y = arctan(-2)



[c] $\tan\frac{1}{2}x$

$= \csc x - \cot x$

$= \dfrac{3}{-\sqrt{5}} - \dfrac{2}{-\sqrt{5}}$

$= -\dfrac{1}{\sqrt{5}}$

$= -\dfrac{\sqrt{5}}{5}$

Prove the identity $\dfrac{\sec^2 y - \csc^2 y}{\tan y - \cot y} = \sec y \csc y$. **SCORE: _____ / 14 PTS**

$$LS = \frac{\cancel{1} + \tan^2 y - (\cancel{1} + \cot^2 y)}{\tan y - \cot y}$$

$$= \frac{\tan^2 y - \cot^2 y}{\tan y - \cot y}$$

$$= \frac{(\tan y + \cot y)\cancel{(\tan y - \cot y)}}{\cancel{\tan y - \cot y}}$$

$$= \tan y + \cot y$$

$$= \frac{\sin y}{\cos y} + \frac{\cos y}{\sin y}$$

$$= \frac{\sin^2 y + \cos^2 y}{\cos y \sin y}$$

$$= \frac{1}{\cos y \sin y}$$

$$= \sec y \csc y$$

Rewrite $\cos^2 x \sin^2 x$ using only the first powers of cosine (and constants and the 4 basic arithmetic operations). **SCORE: _____ / 14 PTS**
Simplify your final answer, which must **NOT** be in factored form, and must **NOT** involve any other trigonometric functions.

$$\frac{1+\cos 2x}{2} \cdot \frac{1-\cos 2x}{2}$$

$$= \frac{1-\cos^2 2x}{4}$$

$$= \frac{1 - \left(\frac{1+\cos 4x}{2}\right)}{4} \cdot \frac{2}{2}$$

$$= \frac{2-(1+\cos 4x)}{8} = \frac{1-\cos 4x}{8}$$

Solve the equation $5 - 2\cos\frac{1}{4}x = 4(1-\cos\frac{1}{4}x)$. **SCORE: _____ / 14 PTS**

$$5 - 2\cos\tfrac{1}{4}x = 4 - 4\cos\tfrac{1}{4}x$$

$$1 = -2\cos\tfrac{1}{4}x$$

$$\cos\tfrac{1}{4}x = -\tfrac{1}{2}$$

$$\tfrac{1}{4}x = \tfrac{2\pi}{3} + 2n\pi \text{ or } \tfrac{4\pi}{3} + 2n\pi$$

$$x = \tfrac{8\pi}{3} + 8n\pi \text{ or } \tfrac{16\pi}{3} + 8n\pi$$